THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1520G/H University Mathematics 2014-2015
Suggested Solution to Assignment 4

Exercise 8.1

(11) $\int \tan^{-1} x dx = x\tan^{-1} x - \int \frac{x}{1+x^2}dx = x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C.$

(23)

$$\begin{aligned}\int e^{2x}\cos 3x dx &= \frac{1}{2}\int \cos 3x de^{2x}\\ &= \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\int e^{2x}\sin 3x dx\\ &= \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}\int e^{2x}\cos 3x dx\end{aligned}$$

Then we find $\int e^{2x}\cos 3x dx = \frac{2}{13}e^{2x}\cos 3x + \frac{3}{13}e^{2x}\sin 3x + C.$

(63) $\int x^n e^{ax}dx = \frac{1}{a}\int x^n de^{ax} = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}dx$

Exercise 8.2

(19)

$$\begin{aligned}\int 16\sin^2 x\cos^2 x dx &= 4\int \sin^2 2x dx\\ &= 2\int 1-\cos 4x dx\\ &= 2x - \frac{1}{2}\sin 4x + C\end{aligned}$$

(21) $\int 8\cos^3 2\theta \sin 2\theta d\theta = -\int 4\cos^3 2\theta d(\cos 2\theta) = -\cos^4 2\theta + C$

(35) $\int \sec^3 x\tan x dx = \int \sec^2 x d(\sec x) = \frac{1}{3}\sec^3 x + C$

Exercise 8.3

(36) Use substitution $t = \ln x, x = \tan\theta$:

$$\begin{aligned}\int_{\ln\frac{3}{4}}^{\ln\frac{4}{3}} \frac{e^t dt}{(1+e^{2t})^{\frac{3}{2}}} &= \int_{\frac{3}{4}}^{\frac{4}{3}} \frac{dx}{(1+x^2)^{\frac{3}{2}}}\\ &= \int_{\tan^{-1}\frac{3}{4}}^{\tan^{-1}\frac{4}{3}} \frac{\sec^2\theta d\theta}{(1+\tan^2\theta)^{\frac{3}{2}}}\\ &= \int_{\tan^{-1}\frac{3}{4}}^{\tan^{-1}\frac{4}{3}} \frac{\sec^2\theta d\theta}{\sec^3\theta}\end{aligned}$$

$$
\begin{aligned}
&= \int_{\tan^{-1}\frac{3}{4}}^{\tan^{-1}\frac{4}{3}} \cos\theta d\theta \\
&= \sin\left(\tan^{-1}\frac{4}{3}\right) - \sin\left(\tan^{-1}\frac{3}{4}\right) \\
&= \frac{4}{5} - \frac{3}{5} = \frac{1}{5}
\end{aligned}
$$